R & D NOTES

Optimal Design of Hollow Fiber Modules

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Membrane separation processes now are recognized as important chemical engineering separation techniques. Several commercial membrane modules are available to carry out purification, separation, and concentration operations. Most of these modules have been designed by an empirical or semiempirical approach. A more rigorous approach to the design of membrane modules is possible if one makes use of our present understanding of the underlying transport mechanisms which enable one to identify the major resistances involved in these systems.

In this note the design features of a hollow fiber module are examined. It is shown that the optimal design of hollow fiber systems can be established in a very simple and straightforward manner.

The starting equations and assumptions which will be used have been discussed in detail by Dandavati et al. (1975). Therefore, only the most pertinent equations will be given. Gill and Bansal (1973) and Bansal and Gill (1974) discussed a method of optimal design for hollow fiber units, but their method is much more complicated than that which will be presented here.

The coordinate system is depicted in Figure 1*a*. The feed is assumed to be pure water so that the permeation velocity through the membrane is given by

$$V_w = A\,[p_1 - p_3(z)] \tag{1}$$

It is reasonable to assume that the pressure on the shell side is roughly constant, and experimental results for the DuPont B-9 permeator support this assumption. However, for large diameter modules, highly viscous fluids, or very high feed flow rates, the shell side pressure drop may be significant. Calculations of the shell side pressure drop and its effect on performance are discussed by Dandavati et al. (1975) and recently by Kabadi (1976).

The pressure drop inside the fiber is described to a good approximation by the following modified Hagen-Poiseuille flow law:

$$\frac{dp_3}{dz} = -\frac{16\,\mu\,r_o}{r_i^4}\int_o^z V_w dz; \quad o \leq z \leq l \tag{2}$$

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$$= -\frac{16\,\mu\, r_o}{r_i^4}\int_o^l V_w dz; \quad l \leq z \leq (l + l_s) \qquad (3)$$

Equations (1) and (3) can be solved easily, and noting that

$$(p_3)_{z=l+l_s} = p_{\text{atm}}$$

one obtains

$$V_w = \frac{A\,\Delta P \cosh\left(\frac{\beta z}{l}\right)}{\cosh(\beta) + \left(\frac{l_s}{l}\right)\beta \sinh(\beta)} \qquad (4)$$

and

$$\frac{p_3 - p_{\text{atm}}}{\Delta P} = l - \frac{\cosh(\beta z/l)}{\cosh(\beta) + \left(\frac{l_s}{l}\right)\beta\sinh(\beta)} \qquad (5)$$

where

$$\beta^2 = \frac{16\,A\,\mu\, r_o l^2}{r_i^4} \qquad (6)$$

In designing a hollow fiber module, one of the objectives is to maximize the permeation flux (F_p) per unit shell volume (V). From the above equations we have

$$\frac{F_p}{V} = S\,\overline{V}_w = \frac{2(1-\epsilon)}{r_o}\;\frac{A\,\Delta P}{\beta\left[\coth(\beta) + \beta\left(\frac{l_s}{l}\right)\right]} \qquad (7)$$

Thus, the explicit parametric dependence of F_p/V can be expressed as

$$\frac{F_p}{V} = f(\epsilon, r_o, r_i, A, \mu, l, l_s, \Delta P) \qquad (8)$$

Equations (7) and (8) are the important equations which may be used to obtain the optimal design of hollow fiber modules. Parameters in these equations should be chosen so as to maximize (F_p/V) for a given operating pressure ΔP. Generally, one has some information about the membrane used, the operating temperature, and the solvent feed. In other words, A and μ are assumed to be known.

The active length l and seal length l_s should be as small as possible to obtain the maximum flux per unit volume of the permeator. Hence, the choices of the active length l and seal length l_s generally are dictated by practical and economic considerations. The porosity of the fiber bundle is somewhat restricted owing to flow distribution and pressure drop considerations. In the present investigation, the porosity will be assumed to be known.

Hollow fibers are thick cylinders, so that they do not require any external support. Therefore, the value of r_o/r_i is chosen to insure the necessary strength requirements, rather than maximum permeate rates. Thus, the specified parameters are

$$\epsilon, r_o/r_i, l, l_s, A, \mu, \Delta P$$

First, let us consider existing modules in which fibers are open at only one end as shown in Figure 1*b*. For fibers open on one end, it is convenient to rewrite Equation (7) in dimensionless form as

$$\psi = \frac{F_p}{V\,\Delta P}\cdot\frac{(r_o/r_i)}{(1-\epsilon)}\cdot\left[\frac{2\mu\, r_o\, l^2}{r_i A^2}\right]^{1/3} = \frac{1}{\beta^{1/3}\left[\coth(\beta) + \beta(l_s/l)\right]} \qquad (9)$$

In Figure 2, ψ is plotted against β and r_i for the following values of A, μ, (r_o/r_i), and l corresponding to the B-9 permeator and water system:

$A = 6.88 \times 10^{-13}\ \text{m}^2\ \text{s/kg}$
$\mu = 10^{-3}\ \text{kg/m}\cdot\text{s}$
$r_o/r_i = 2$
$l = 0.75\ \text{m}$
$l_s/l = 0.1$

As r_i increases, the pressure drop inside the fiber will decrease and consequently the permeation rate will increase. However, increasing the fiber size will reduce the surface area per unit volume, and hence the total permeation rate will decline. The optimum fiber bore radius of 21μ is exactly equal to that selected by the investigators at DuPont.

It can be seen from Equation (9) that for a given l, increasing the seal length l_s, will reduce the dimensionless permeation rate per unit volume ψ. Also, the optimum value of β and the fiber bore radius will be affected by changing l_s.

In order to see the effect of various parameters on the optimum fiber bore radius, Equation (7) is differentiated with respect to r_i, and the result is equated to zero:

$$\frac{\partial}{\partial r_i}\left(\frac{F_p}{V\,\Delta P}\right) = 0, \text{ for constant values of } \epsilon, \frac{r_o}{r_i}, A, \mu, l, l_s$$

The result for fibers open at one end is

$$\coth(\beta_o) + 4\beta_o\left(\frac{l_s}{l}\right) - 3\beta_o \operatorname{cosech}^2(\beta_o) = 0 \qquad (10)$$

The optimum value of β (one end open), denoted β_o, is plotted as a function of (l_s/l) in Figure 3. It is known that any increase in the membrane permeability A, solvent viscosity μ, active fiber length l, or the seal length l_s will be accompanied by an increase in the pressure drop inside the fiber. In order to obtain maximum permeation rate for a given shell volume and applied pressure, an increase in A, μ, l, or l_s will require more internal flow area; that is, r_i will have to be increased. It may be concluded from Figure 3 that the pressure drop inside the fiber is an important rate limiting factor.

In view of the above observations, it appears that any attempt to improve the efficiency of a hollow fiber module should be focused upon decreasing the pressure drop inside the fiber. For example, this suggests that one should study the effect of having fibers open at both ends (see Figure 1*c*) as opposed to the present practice of using fibers closed at one end and open at the other end. Obviously, one must ask if the increased permeate rate per unit shell volume at a given pressure is high enough to justify any increase which may occur in the manufacturing cost. To answer this question, the above analysis is extended to calculate the increase in the permeate rate.

In terms of the new coordinate

$$z_1 = \frac{l}{2} - z \qquad (11)$$

the boundary condition and Equation (3) become

$$(p_3)_{z_1 = \pm\left(\frac{l}{2} + l_s\right)} = p_{\text{atm}} \qquad (12)$$

$$\left.\begin{aligned} \frac{\partial p_3}{\partial z_1} &= -\frac{16\mu\, r_o}{r_i^4}\int_o^{z_1} V_w\, dz_1;\ 0 \leq |z_1| \leq l/2 \\ &= -\frac{16\mu\, r_o}{r_i^4}\int_o^{l/2} V_w\, dz_1;\ l/2 \leq |z_1| \leq \frac{l}{2} + l_s \end{aligned}\right\} \qquad (13)$$

The above equations along with Equation (1) can be solved to obtain the permeation rate when fibers are open at both ends:

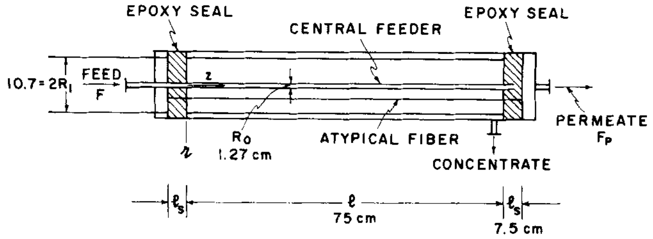
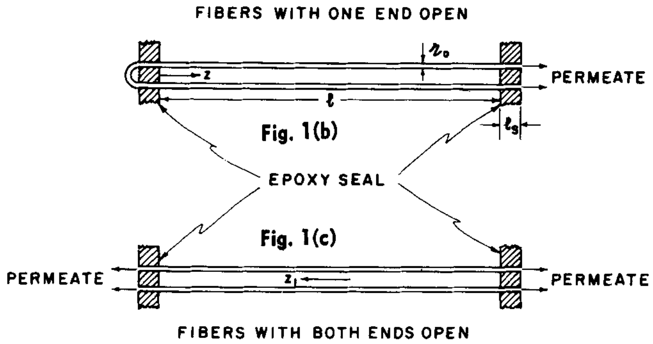


Fig. 1(a). Schematic diagram and coordinates for radial flow systems.



(b). Fibers with one end open. (c). Fibers with both ends open.

$$\psi = \frac{F_p}{V\,\Delta P} \cdot \frac{(r_o/r_i)}{(1-\epsilon)} \cdot \left[\frac{2\mu\, r_o\, l^2}{r_i\, A^2}\right]^{1/3}$$

(both ends open)

$$= \frac{2}{\beta^{1/3}\left[\coth\left(\frac{\beta}{2}\right) + \beta \frac{l_s}{l}\right]} \tag{14}$$

Equation (14) is plotted in Figure 2. It is clear that having open ended fibers reduces the pressure drop inside the fibers and improves the productivity significantly. Indeed, the maximum permeation rate in an open ended fiber bundle is 47% greater than the maximum rate in the units presently available. Also, in case of the open ended fibers, the optimum fiber bore radius is 14.5μ. If fibers with a bore radius of 21μ are used, the productivity can be increased by 31% if one opens both ends of the fibers.

The optimizing equation for the case of open ended fibers can be derived in a similar manner to that used for Equation (1), and one obtains

$$\coth(\beta_o/2) + 4\,\beta_o\,(l_s/l) - 3\,(\beta_o/2)\,\mathrm{cosech}^2\,(\beta_o/2) = 0 \tag{15}$$

(both ends open)

Equation (15) is plotted in Figure 3. When $l_s/l = 0.1$, for example, the optimum value of β_o is changed from 1.155 for the uniflow fibers (permeate flows from the closed end to the open end) to 2.04 in case of open ended fibers.

It can be shown from Equations (9) and (10) that in the case of fibers with one end open, the maximum possible value of ψ is 0.79 when $l_s/l = 0$ and the corresponding value of β_o is 1.42. Similarly, in the case of fibers with both ends open, Equations (14) and (15) indicate that the maximum value of ψ is 1.25 for $l_s/l = 0$ and $\beta_o = 2.84$. Thus, the permeation rate per unit volume ψ in this case will increase by about 58%.

CONCLUSION

The design procedure presented here shows that the DuPont B-9 permeator has been designed to produce optimum permeation rates per unit shell volume at a given applied pressure. However, its performance can be improved by 47% if the fibers are open at both ends and if the fiber bore radius is 14.5μ. Even when existing fibers of 21μ bore radius are used, the permeation rate can be increased by 31% by opening the closed end of the fibers.

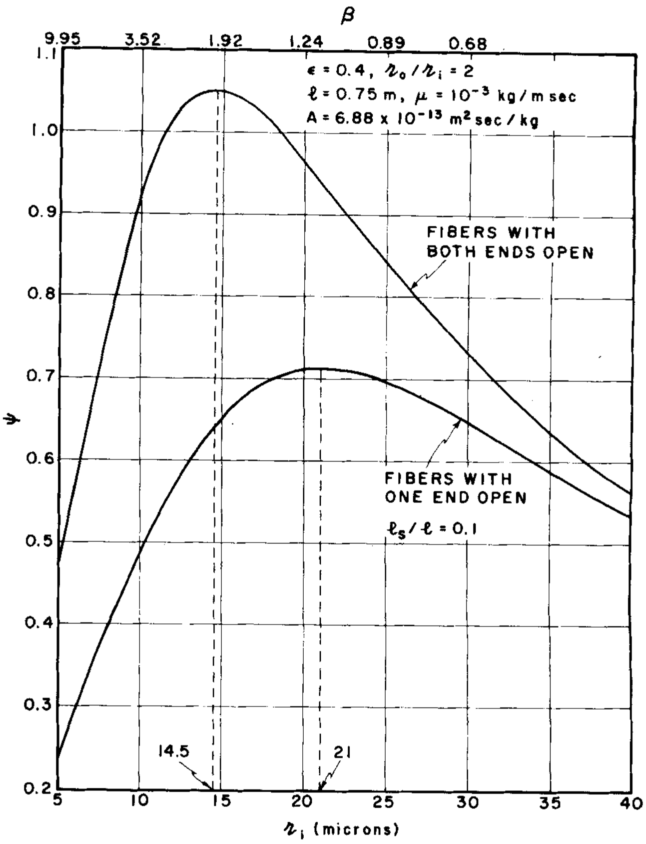


Fig. 2. Variation of dimensionless permeation rate per unit volume with fiber radius.

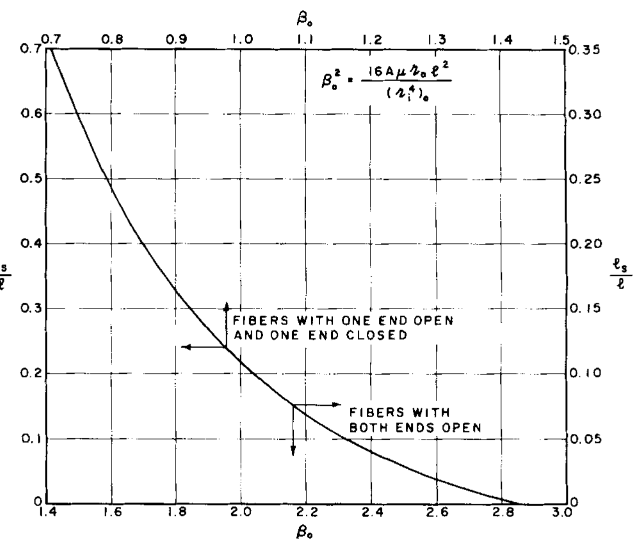


Fig. 3. Optimum value of β as a function of (l_s/l).

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NOTATION

A = membrane solvent permeability constant, m^2 s/kg
F_p = permeability rate, m^3/s
l = active length of fiber, m
l_s = seal length, m
p_1 = pressure in the shell, N/m^2

p_3 = pressure in the fiber, N/m^2
p_{atm} = pressure at the open end of fiber, N/m^2
ΔP = $p_1 - p_{atm}$
r_i = fiber bore radius, m
r_o = outside fiber radius, m
S = surface area of fibers per unit shell volume, m^2/m^3
V = shell volume, m^3
V_w = permeation velocity, m/s
$\bar{V}_w$ = length averaged permeation velocity, m/s
z = axial coordinate measured from the closed end of a fiber, m
z_1 = axial coordinate measured from the midpoint of a fiber, m

Greek Letters

β = defined by Equation (6)
β_o = optimum value of β
ϵ = void fraction
μ = solvent viscosity, kg/m·s
ψ = defined by Equation (9)

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Knudsen Diffusion Through Noncircular Pores: Textbook Errors

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Eldridge and Brown (1976) state that: "For Knudsen flow, while the cylindrical pore formula derivation is presented in many places, the effect of a noncircular cross section on the transport rate does not appear to have been considered quantitatively." They then present the results of their numerical calculation for the Knudsen diffusion coefficient (D_K) in pores of rectangular and elliptical cross section of varying aspect ratio. In fact, the problem of the effect of geometry on the Knudsen diffusion coefficient has been treated in the past, though not always correctly (Knudsen, 1909; Smoluchowski, 1910; Loeb, 1934; Dushman, 1962); it is the purpose of this note to single out the error which appears in these references and to indicate an efficient, numerical procedure for the accurate calculation of D_K for arbitrary pore geometry. Moreover, an incorrect equation initially obtained by Knudsen has been used inadvertently in two recent investigations involving diffusion through rhomboidal pores (Quinn et al., 1972; Beck, 1969), while in other studies no correction for the noncircular cross section has been made (Ho, 1971; Petzny and Quinn, 1969). We have computed the correct results for this case and discuss the error involved in these studies wherein pore size was deduced from measured Knudsen flux.

The following formula was derived by Knudsen (1909) for the flow of a rarefied gas through a long cylindrical tube of arbitrary cross section:

$$N = \frac{4}{3} \frac{\bar{v}}{\int_0^L \frac{H}{A^2} dL} \frac{(P_2 - P_1)}{R_o T} \tag{1}$$

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This result follows from equating the momentum transferred to the pore wall by colliding gas molecules to the force acting on the gas due to the pressure difference across the pore. If the Knudsen diffusion coefficient is defined as the ratio of the molar flux to concentration gradient along the pore, then the above equation gives

$$D_K = \frac{4}{3} \bar{v} \left(\frac{A}{H} \right) = \frac{4}{3} \bar{v} \, r_h \tag{2}$$

where r_h is simply the hydraulic radius. For the circular pore ($r_h = r/2$), Knudsen's expression yields the familiar result. Although Equation (1) is cited in standard references (Loeb, 1934; Dushman, 1962), it is, in fact, correct only for a circular pore.

Equation (2) has been employed in connection with measurements of Knudsen flow in 100 to 1 000Å pores created in thin ($\sim 7\,\mu$) mica sheets by an irradiation-etching technique. Briefly, the technique consists of bombarding mica sheets with massive fission fragments from a radioactive source and etching material from the resulting damage tracks with acid. Model pores with parallel walls are formed which are rhomboidal in cross section (the included angles being 60 and 120 deg). Pore size has been determined by two independent means (by measuring Knudsen flow through gas filled pores and by monitoring conduction of electrolytes through solution filled pores), and these have been compared on the basis of an equivalent pore radius defined as that of a circle having area equal to that of the rhomboidal pore (Quinn et al., 1972):

$$r = \left(\frac{\sqrt{3}}{2\pi} \right)^{1/2} w \tag{3}$$